

APPLICATION OF PARAMETER STATISTICAL TESTS AND DATA ENVELOPMENT ANALYSIS METHODS IN MODERN BUSINESS

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Abstract: In the absence of a sufficient amount of information for quality business decision-making, i.e. successful performance of activities without unnecessary losses in the consumption of inputs, recently the non-parametric DEA method (Data Envelopment Analysis) is most often used through the linear programming technique. In the event that company managers have enough information to make business decisions, parametric statistical tests are used that compare the company's current performance with optimal performance, i.e. those that are on the edge of efficiency. However, this situation is very rare, so before making business decisions, non-parametric and then parametric statistical tests are carried out in detail. The subject of research of this paper is primarily focused on the simultaneous application of parametric and non-parametric statistical tests in the assessment of the economic efficiency of an economic entity.

After the conducted research and analysis of the obtained results, it was determined that the null hypothesis, which claims that the relative efficiency of the warehouse obtained by parametric statistical tests and the DEA method is identical and that the trends have the same direction, could not be fully accepted. Namely, it was found out that the results of one and the same economic situation using the mentioned two types of analysis differ to the extent that they are not adequate for economic decision-making, however identical results were obtained in the assessment of the trend. It can be concluded that the simultaneous application of both methods, as well as its implementation in several iterations, can provide enough quality information for effective decision-making. Stochastic processes that occur during the implementation of business decisions using the DEA technique can be minimized through the simultaneous application of statistical parametric methods and tests for evaluating the expected efficiency of DEA. The effectiveness of this method in any case depends on the size of the sample implemented in the aforementioned statistical analysis. The aforementioned statistical tests enable the measurement and detection of those input parameters that will most effectively contribute to the efficiency of business systems.

Keywords: DEA, parametric statistical tests, linear programming, sample, business processes, information.

Field: Economy

1. INTRODUCTION

The stochastic processes that occur during the implementation of business decisions using DEA methods can be minimized through the simultaneous application of statistical parametric methods and tests to assess the expected effectiveness of DEA. The effectiveness of this method in any case depends on the size of the sample that is implemented in the aforementioned statistical analysis. The larger this sample, the more reliable the result of statistical parametric tests (Anđelković, 2022).

On the other hand, the inclusion of a larger sample in the statistical analysis increases the costs of its implementation and also requires a longer time for its implementation. Choosing the required sample size for statistical analysis requires a careful and multidisciplinary approach.

In any case, the simultaneous use of non-parametric and parametric statistical tests is the most effective tool for making quality business decisions. Parametric statistical tests, which evaluate the set hypotheses in DEA analysis, contribute a lot to the reliability of management functions.

The aim of the simultaneous application of non-parametric and parametric methods contributes to a large extent to quality management of business processes, efficiency and profitability of business systems (Deakin, 2001).

The necessity of using parametric statistical tests in the evaluation of the applied DEA methods is necessary in situations where there is an insufficient amount of relevant information available that can lead to incorrect conclusions when making business decisions and thus the deviation of the business system from the optimal business trajectory with incalculable negative consequences for that business system.

The same is the case in situations where business decision-makers have too much information

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about an economic phenomenon, so the selection of the right and adequate information requires the combination of parametric statistical tests and DEA methods (Glišović, 2017).

2. PARAMETER STATISTICAL TESTS

When applying parametric statistical tests in practice, one must take into account the fact that the parameter represents the size of the overall observed set, while statistical analysis usually operates with the concept of a sample of a statistical set, which may or may not represent the objective result of the entire observed phenomenon.

These tests assume that the random variable will have a normal probability distribution.

Unlike parametric statistical tests, non-parametric tests operate exclusively on the values of the entire observed set (Jiyoung, 2019).

A nonparametric statistical test is a test that allows two sets of conditions or treatments to be compared without assuming that the values are normally distributed. Thus, for example, one can compare the speed with which two different groups of people can run 100 meters, where one group trained for two months and the other did not.

Requirements typically relate to the following:

- Two random, independent samples;
- Data are continuous - in other words, in principle it must be possible to distinguish values at the n th decimal place;
- The measurement scale should be ordinal, interval or proportional;
- For maximum accuracy, there should be no ties, although this test - like others - has a way to resolve ties.

Null hypothesis. The null hypothesis claims that the median of the results obtained by the parametric statistical tests and the DEA method is identical.

The basic equation can be written as,

$$U = NM + \frac{N(N+1)}{2} - \sum_{x_i} \text{Rank}(x_i)$$

To solve some of the problems, appropriate parametric equations are introduced, where instead of directly defining the function from the variable $f(x)$, both the function and the variable are defined in terms of a third variable called a parameter, e.g. like,

$$x=f(t)g=g(t)x=f(t)g=g(t)$$

Each value of (t) defines a point,

$$(x,g) \Rightarrow (f(t),g(t))(x,g)=(f(t),g(t)).$$

Plotting a parametric curve is not always straightforward. There are often parameter restrictions and this affects the form of parametric equations or functions (Međurečan, 2019).

3. STATISTICAL TESTS USING DEA METHODS

DEA analysis has been proven to be an excellent data-driven efficiency analysis method for comparing multiple-input, multiple-output decision-making units (DMUs). In the classic DEA concept, it is assumed that the status of each measure is clearly displayed (hence known) as an input or an output. However, in some situations, a performance measure (the efficiency of one DMU i) may play an input role for some DMU j and an output role for other DMU k (Pjanić, 2016).

The classification of inputs and outputs is done at the level of variables that are labeled as flexible measures.

The example of higher education institutions in the UK demonstrates the applicability of the DEA-based approach mentioned here. In this case, DEA analysis is used to examine the technical efficiency (TE) of 45 British higher education institutions (universities) in the period 1980/81–1992/93. This period saw major changes in public funding and the extent of student and staff participation.

In order to shed light on the causes of efficiency variations, TE is decomposed into purely technical efficiency (BCC), congestion efficiency and volume efficiency. The analysis showed that there was a significant increase in the weighted geometric mean of TE scores during this period under investigation, although this increase was most visible between 1987/1988 and 1990/1991.

The rising results of TE are mainly attributed to good results in pure technical efficiency and congestion efficiency, with volume efficiency playing a minor role.

A Malmquist approach is then used to distinguish changes between TEs and intertemporal shifts in the efficiency frontier.

The results reveal that total factor productivity increased by 51.5% between 1980/1981 and 1992/1993, and that most of this increase was due to a significant shift beyond the efficiency frontier during that period.

DEA analysis of the efficiency of higher education institutions was also applied on the example of Selcuk University. In this study, an attempt was made to determine how effectively and efficiently the institutions work with the used input and output parameters (total costs, number of graduates, average academic grades). All this was accomplished by the faculty affiliated with Selcuk University (Radosavljević, 2022).

It is important to inform management and create insight into the decisions that need to be made in accordance with changing factors for the future. In the subject study, the annual report of the administration of Selcuk University for 2018 and data on the results of the academic incentive for 2018 were used. DEA, as a non-parametric technique, used the principles of linear programming in the analysis of relative efficiency in order to enable the mutual comparison of institutions or DMUs.

As a result of the study, an analysis of the relative efficiency of the units was performed and evaluations were made regarding the ways in which the inputs and outputs were not efficient, and the identification of ways to make them effective. If it is effective, the capacity would be kept at the level of existing activities, and if it is not, then for such unused capacity, estimates of the unused capacity were made (with proposed measures for their more successful operation) (Radosavljević, 2022).

At the level of the aforementioned cases, DEA models were developed to calculate the technical efficiency of DMUs with flexible measures. For these types of systems, the conventional DEA model is modified to include flexible measures.

The proposed approach is potentially useful in many applications, including the evaluation of production efficiency, health care systems, educational institutions where a general variable in a flexible DEA structure can be considered an input or an output.

The analysis began with an axiomatic study of the proposed model. Finally, an application in higher education was used to demonstrate the usefulness of the subject model. Further research is mainly focused on the transformation of other DEA-based models, e.g. the bottleneck-based model, where a similar concept is used (Shahidul, 2020).

The flexibility model based on DEA can be applied in a wider and different range of companies, basically at the level of various processes in the organization, but not at the level of the entire organization (also models: ABC, Brownow model, DtM, KBEMS).

In processes, operations, quality and excellence, different quality trends are present. Some of the concepts that have experienced intensive application are highlighted here, such as: TQM (introduced in the early 1990s), Six Sigma, ISO 9000 and other HSE standards, TPM in production, BPR, TOC, excellence frameworks such as EFQM, Lean, Lean Six Sigma integration (Santosh Govind Bhosale, 2015).

The theory of constraints (TOC - Theory of constraints) is a management paradigm that considers every controllable system as limited in achieving a number of its goals (although with a very small number of constraints).

This means that TOC always implies that there is at least one constraint in the firm. TOC uses a focusing process to identify a constraint and restructure the rest of the organization around it. TOC also adopts the common thesis that "a chain is no stronger than its weakest link". This means that organizations and processes are vulnerable because the weakest DMU or part can always damage, threaten or force them into bankruptcy and liquidation, or at least adversely affect the business outcome (Simonović, 2019).

In some works, when applying DEA, an alternative model is proposed in which each flexible measure is treated as an input or output variable.

The main interest of the DEA method is to make the business system as efficient as possible and to evaluate the technical efficiency as well as possible.

An illustration of the proposed model with application in higher education institutions in the UK provides a real example of the proposed model with a flexible structure. The data used by (Simonović, 2019) are applied. Two factors were selected as inputs: general expenditure (k_1) and equipment costs (k_2), and three factors as outputs: undergraduate students (i_1), postgraduate research (i_2) and postgraduate teaching (i_3). The flexible measure here was research income (z_1). The research income status for each university was determined by the same DEA model, while the data set consists of 50 universities.

Model results indicate that research income was considered an input or output variable in the estimation model. It was observed that 25 universities (out of 50 analyzed) treat research income as an output

measure, and 25 universities treat it as an input measure.

Researched efficiency of higher education institutions in two cases, using DEA-CCR model, one with research income as input and another model with research income as output, shows that research income should be included in the output.

4. STATISTICAL FOUNDATION FOR DEA

DMUs, in these considerations, contain an output vector and an input vector, where $i \in I$ and $k \in K$ s are convex subsets of RR and RI , respectively. The input quantities and the variable proportions of the output mixture are random variables (Šegrt, 2022).

The DEA estimator is defined in accordance with the basic definition (such as: an estimator - who makes an estimate; or - in statistics: a rule, method or criterion for achieving an estimate of a parameter's value). Of course, in other cases it can also represent a person who assesses the value, price, number, quantity or volume of something.

DEA methods effectively determine the efficiency of the business system independently of the input parameters. By using these methods, a greater number of system solutions can be reached, which can be efficiently implemented with the effective use of DEA and null hypothesis tests.

These methods and tests can easily and transparently determine whether the set parameters in the null hypothesis are underestimated or overestimated. Just by knowing whether the initial parameters of the hypothesis are greater or less than the results of this analysis, it is possible to correct the value of the null hypothesis and then re-evaluate it.

We note here that the experience of the person making the assessment plays a big role in setting the value of the null hypothesis. And finally, the use of information technologies plays an important role in the preparation and processing of data, which as a result will have valid parameters for establishing the null hypothesis.

5. HYPOTHESIS TESTS FOR SITUATIONS CHARACTERIZING SHIFTS AT THE BORDER

In situations where the necessary data are available in the research of changes in the profitability of market participants in a certain time interval, all the necessary hypothesis test procedures must be foreseen beforehand in order to apply them in the most efficient way (Springer, 2016).

In the specific case, we can consider all variations, around the estimated values of the set hypotheses in situations characterized by shifts at the border, as non-deterministic or random quantities.

The subject tests are rarely applied in situations where we do not have a large enough sample and in these situations they should be used with great caution. Additional simulation studies are always warranted to provide valid evidence of small sample performance (Marjanović, 2018).

The use of information technologies as well as the modernization of statistical tests in DEA in any case contribute to greater accuracy of their results and facilitate decision-making by the competent management. The development and expansion of methods and appropriate tests in this analysis is a constant and continuous process (Marjanović, 2018).

Type references should also be taken into account, as a certain complementarity can be observed as in the case of the analysis of problems with flexible flow stores analyzed using DEA (Springer, 2016).

6. RESEARCH RESULTS AND DISCUSSIONS

On a simple example, we will analyze the application of the DEA technique and the application of parametric statistical tests to assess the efficiency of a business system in the specific case of a store, depending on the invested capital and realized net profit. Furthermore, our goal is to compare the obtained results of both approaches and determine their possible deviations.

Null hypothesis. The null hypothesis states that the relative efficiency of the store obtained by parametric statistical tests and the DEA method is identical and that the trends have the same direction.

In the following example, the relative efficiency of the store and the trend depending on capital and net profit was calculated using parametric statistical tests (Table 1) as well as using DEA methods (Table 2).

Table 1: Efficiency and trend obtained using parametric statistical tests

Capital	Net gain	% profitability	Eff	Trend
3,636	0,2667	7,3	0,883	↗
1,832	0,3333	18,19	0,981	↘
1,743	0,1667	9,56	0,709	↗
1,654	0,1543	9,33	0,941	↘
1,598	0,1442	9,00	0,842	↘

Source: The authors

Table 2: Efficiency and trend obtained using DEA methods

Capital	Net gain	% profitability	Eff	Trend
3,636	0,2667	7,3	0,502	↗
1,832	0,3333	18,19	0,943	↘
1,743	0,1667	9,56	0,472	↗
1,654	0,1543	9,33	0,536	↘
1,598	0,1442	9,00	0,502	↘

Source: The authors

It can be observed that the trends clearly have the same direction, and the maximum efficiency level is also similar with minimal deviations. The estimated maximum efficiency for the capital level of 1,832 stores using parametric statistical tests is 3.8% higher than the same estimate made using the DEA method. It can also be seen that in addition to the maximum effectiveness estimates, with this level of capital, the highest percentage of profitability was achieved.

Table 3: Comparative overview of effectiveness assessment by statistical and DEA method

Capital	Eff estimated by statistical method	Eff estimated DEA method	Difference	% difference
3,636	0,883	0,502	+ 0,381	56,85
1,832	0,981	0,943	+ 0,038	3,8
1,743	0,709	0,472	+ 0,237	66,57
1,654	0,941	0,536	+ 0,405	56,96
1,598	0,842	0,502	+ 0,34	59,62

Source: The authors

Table 3 shows a large difference in the assessment of store efficiency for other levels of capital and profit determined by these two methods. This difference is smaller in all observed cases when it was performed using the DEA method ($E(\text{ff.DEA}) < E(\text{ff.PST})$). The differences are huge.

The causes of the mentioned differences can be primarily attributed to the stricter criterion for the calculation of efficiency in DEA methods, the application of an insufficient number of parameters, as well as the size of the observation sample.

The null hypothesis put forward when approaching this analysis is partially rejected because the results of the relative efficiency of the store estimated by the mentioned methods are too different to reach a valid conclusion. The second part of the hypothesis is accepted because the trends in both methods match.

7. CONCLUSION

In the absence of a sufficient amount of information for quality business decision-making, i.e. suc-

successful performance of activities without unnecessary losses in the consumption of inputs, recently the non-parametric DEA method (English Data Envelopment Analysis) is most often used, which is carried out through the linear programming technique.

In the event that the managers of the companies have enough information to make business decisions, parametric statistical tests are used, which compare the current performance of the company with the optimal performance, i.e. those that are at the efficiency limit.

Statistical tests make it possible to measure and detect those input parameters that will most effectively contribute to the efficiency of business systems.

Stochastic processes that occur during the implementation of business decisions using the DEA technique can be minimized through the simultaneous application of statistical parametric methods and tests to assess the expected effectiveness of DEA. The effectiveness of this method in any case depends on the size of the sample that is implemented in the aforementioned statistical analysis. The larger this sample, the more reliable the result of statistical parametric tests.

In any case, the simultaneous use of non-parametric and parametric statistical tests is the most effective tool for making quality business decisions. Parametric statistical tests, which evaluate the set hypotheses in the DEA analysis, greatly contribute to the reliability of management functions.

The aim of the simultaneous application of non-parametric and parametric methods contributes to a large extent to quality management of business processes, efficiency and profitability of business systems.

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